Closed Loop Control of a Type 2 Single Pole Sysem

Define the plant and controller, calculate the closed loop gains.

$$G_a(s) = \frac{K}{s^2}$$

Type 2 single pole system

$$G_c(s) = \frac{K_c \cdot (s + sZero)}{s + sPole}$$

Standard controller PID in the s domain.

$$CLTF(s) = \frac{\frac{K_c \cdot (s + sZero)}{s + sPole} \cdot \frac{K}{s^2}}{1 + \frac{K_c \cdot (s + sZero)}{s + sPole} \cdot \frac{K}{s^2}}$$

Closed loop transfer function (CTLF) for combined actuator and controller transfer functions.

$$CLTF(s) = \frac{G_c(s) \cdot G_a(s)}{1 + G_c(s) \cdot G_a(s)}$$

Closed loop transfer function with the characteristic equation in the denominator.

$$CLTF(s) = \frac{K_c \cdot (s + sZero) \cdot K}{s^3 + s^2 \cdot sPole + K_c \cdot K \cdot s + K_c \cdot K \cdot sZero}$$

$$(s + \lambda)^3$$

Desired characteristic equation. In this case a critically damped response is desired.

Find the difference between the system characteristic equation and the desired characteristic equation and the solve for the gains.

$$DiffCE := s^3 + s^2 \cdot sPole + K_c \cdot K \cdot s + K_c \cdot K \cdot sZero - (s + \lambda)^3$$

$$DiffCE \left| \begin{array}{l} \text{coeffs}, s \\ \text{solve}, \begin{pmatrix} K_c \\ sPole \\ sZero \end{pmatrix} \\ \text{simplify} \end{array} \right. \rightarrow \begin{pmatrix} 3 \cdot \frac{\lambda^2}{K} & 3 \cdot \lambda & \frac{1}{3} \cdot \lambda \end{pmatrix}$$

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Define System Parameters

$K := 10$ Millimeters per second squared per % control output

Define the desired response moving the poles on the negative real axis..

$\lambda := 2 \cdot \pi \cdot 10$ $\lambda = 62.831853$

Calculate Lead Lag Filter Gains

$K_c := 3 \cdot \frac{\lambda^2}{K}$ $K_c = 1184.352528$

$sPole := 3 \cdot \lambda$ $sPole = 188.495559$

$sZero := \frac{1}{3} \cdot \lambda$ $sZero = 20.943951$

Convert controller's pole and zero in the s domain to the z domain.

Use matched z transform. $(s + \alpha) = 1 - z^{-1} \cdot e^{-\alpha \cdot T}$

$\frac{K_c \cdot (s + sZero)}{s + sPole}$ becomes $\frac{K_d \cdot \left(1 - zZero0 \cdot z^{-1}\right)}{1 - zPole \cdot z^{-1}}$

Matched z transforms have advantages over using other convsersion methods like Tustin's approximation.

$T := 0.001$ Controller update period

$zPole := \exp(-sPole \cdot T)$ $zPole = 0.828204$ $zZero := \exp(-sZero \cdot T)$ $zZero = 0.979274$

Calculate the discrete controller gain

$K_z := \frac{K_c \cdot (1 - zPole)}{1 - zZero}$ $K_z = 9816.913195$

The gain for the difference equation must be adjusted so the steady state gain remains the same. z is 1 at steady state.

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Simulation

$A_c := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\qquad$ $A_c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$B_c := \begin{pmatrix} 0 \\ K \end{pmatrix}$ $\qquad$ $B_c = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$

$C := (1 \quad 0)$ $\qquad$ $D := 0$

$I := \mathrm{identity}(2)$ $\qquad$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

The formula for converting state space to a Laplace transform transfer function

$C \cdot (s \cdot I - A_c)^{-1} \cdot B_c + D \rightarrow \dfrac{10}{s^2}$ $\qquad$ This should be the plant transfer function if the state space arrays are set up right.

$T = 0.001$ $\qquad$ Update interval

Calculate arrays for use in discrete time.

$$A := I + \sum_{n=1}^{7} \frac{(A_c \cdot T)^n}{n!} \qquad B := \left[I + \sum_{n=1}^{7} \frac{(A_c \cdot T)^n}{(n+1)!} \right] \cdot B_c \cdot T$$

$$A = \begin{pmatrix} 1 & 0.001 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 5 \times 10^{-6} \\ 0.01 \end{pmatrix}$$

Closed Loop Control of a Type 2 Single Pole Sysem

Lead/Lag Control

$x_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Intial position and velocity

$u_0 := 0$ Initial control output

$err_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Initial error

$$LL(r, x, u, err) := \left| \begin{array}{l} x_1 \leftarrow A \cdot x + B \cdot u \\ err \leftarrow \begin{pmatrix} r_0 - x_0 \\ err_0 \end{pmatrix} \\ u_1 \leftarrow \max\left[\min\left[zPole \cdot T \cdot u + K_z \cdot \left(err_0 - zZero \cdot err_1\right), 100\right], -100\right] \\ \begin{pmatrix} x_1 \\ u_1 \\ err \end{pmatrix} \end{array} \right.$$

$N := \frac{1}{T}$ $\quad n := 0 .. N$ Simulate 1 second

$$r_n := \left| \begin{array}{ll} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \text{if } n < 0.01 \cdot N \\ \begin{pmatrix} 2 \\ 0 \end{pmatrix} & \text{if } 0.01 \cdot N \le n \wedge n < 0.3 \cdot N \\ \begin{pmatrix} -2 \\ 0 \end{pmatrix} & \text{if } 0.3 \cdot N \le n \wedge n < 0.7 \cdot N \\ \begin{pmatrix} -1 \\ 0 \end{pmatrix} & \text{otherwise} \end{array} \right.$$

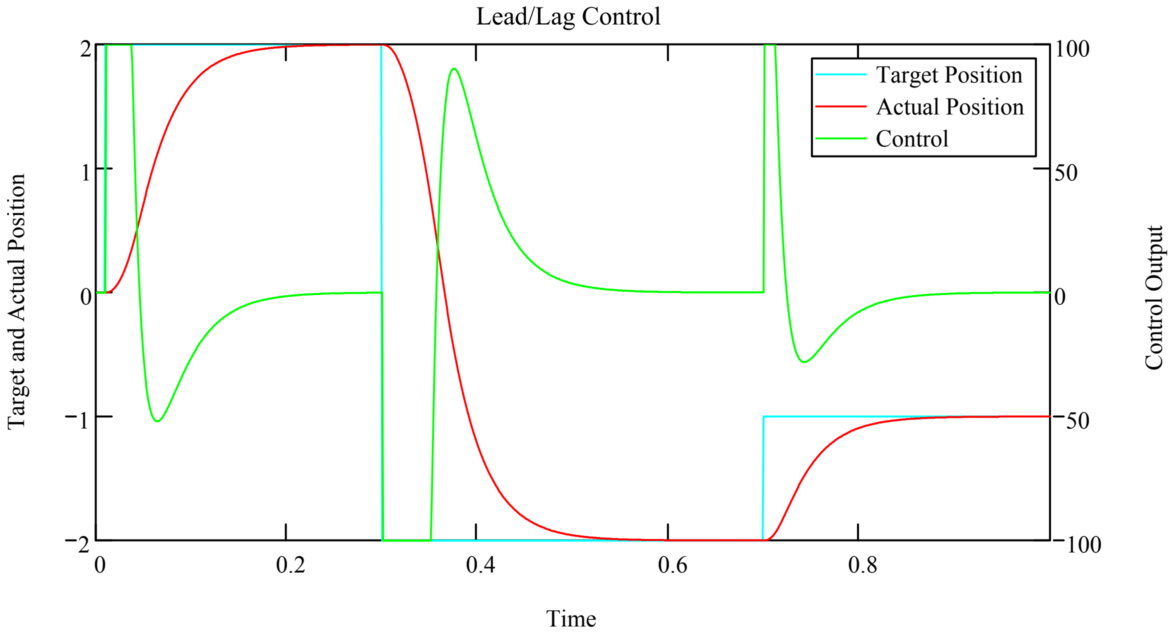
Calculate the target positions and velocities. In this case the target velocities are 0 since that target positions is making step jumps. I like to test how the controller handles output saturation.

$$\begin{pmatrix} x_{n+1} \\ u_{n+1} \\ err_{n+1} \end{pmatrix} := LL\left(r_n, x_n, u_n, err_n\right)$$

Compute the system response and control output

Closed Loop Control of a Type 2 Single Pole Sysem

Verify By Simulating and Graphing the Results.



Closed Loop Control of a Type 2 Single Pole Sysem

Lead/Lag Control of a Constand Acceleration Ramp

$acc := 250$

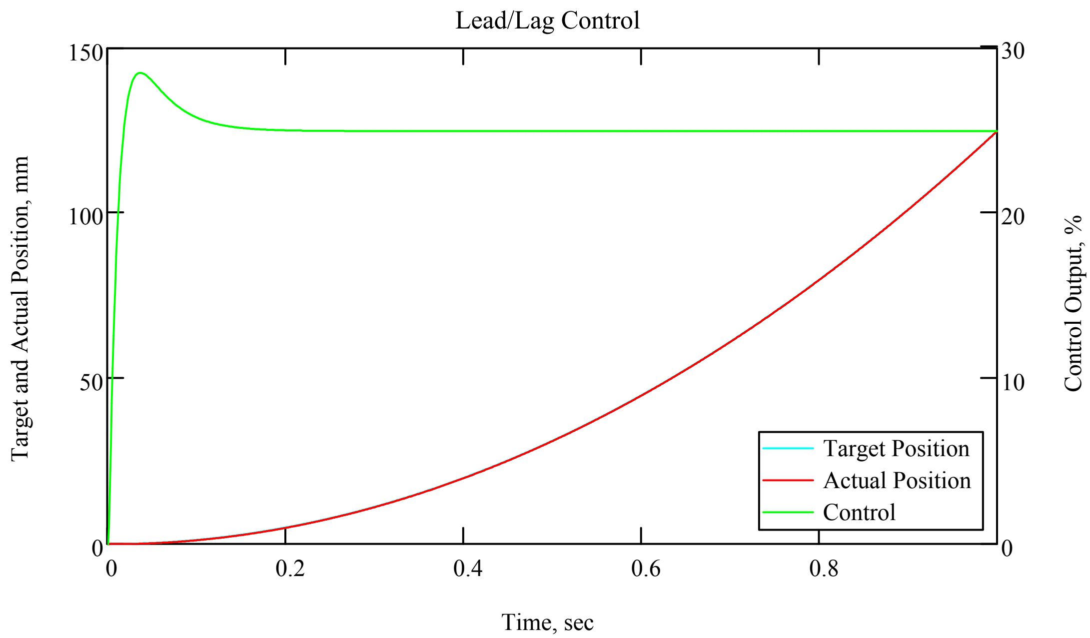
Accelerate at 250 mm/sec^2

$$r_n := \begin{bmatrix} \frac{acc}{2} \cdot (n \cdot T)^2 \\ acc \cdot n \cdot T \end{bmatrix}$$

Calculate the target positions and velocities for a constant acceleration ramp.

$$\begin{pmatrix} x_{n+1} \\ u_{n+1} \\ err_{n+1} \end{pmatrix} := LL\left(r_n, x_n, u_n, err_n\right)$$

Compute the system response and control output



Closed Loop Control of a Type 2 Single Pole Sysem

Lead/Lag Control of a Sine Wave

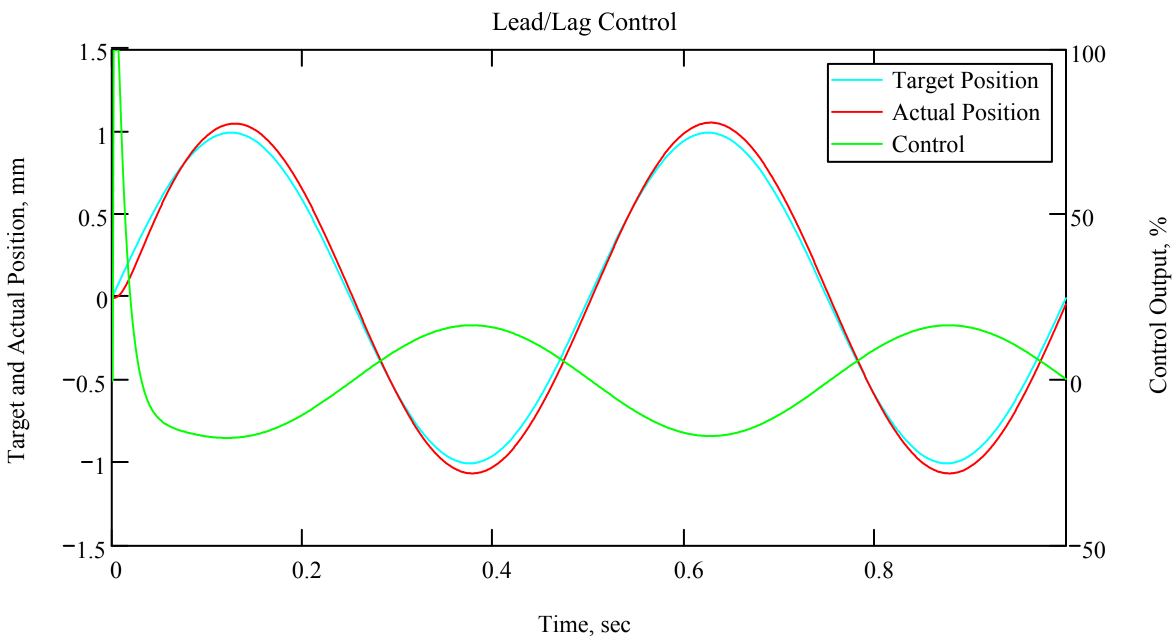
$\text{Amp} := 1 \qquad \text{Hz} := 2$

$$r_n := \begin{pmatrix} \text{Amp} \cdot \sin(2 \cdot \pi \cdot \text{Hz} \cdot n \cdot T) \\ 2 \cdot \pi \cdot \text{Hz} \cdot \text{Amp} \cdot \cos(2 \cdot \pi \cdot \text{Hz} \cdot n \cdot T) \end{pmatrix}$$

Calculate the target position and velocities for a sine wave.

$$\begin{pmatrix} x_{n+1} \\ u_{n+1} \\ err_{n+1} \end{pmatrix} := LL(r_n, x_n, u_n, err_n)$$

Compute the system response and control output



The control output looks like it is 180 degrees out of phase with the positions.

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Closed Loop Bode Plot Calculations

$$\mathrm{CLTF}(s) := \frac{\dfrac{K_c \cdot (s + \mathrm{sZero})}{s + \mathrm{sPole}} \cdot \dfrac{K}{s^2}}{1 + \dfrac{K_c \cdot (s + \mathrm{sZero})}{s + \mathrm{sPole}} \cdot \dfrac{K}{s^2}}$$

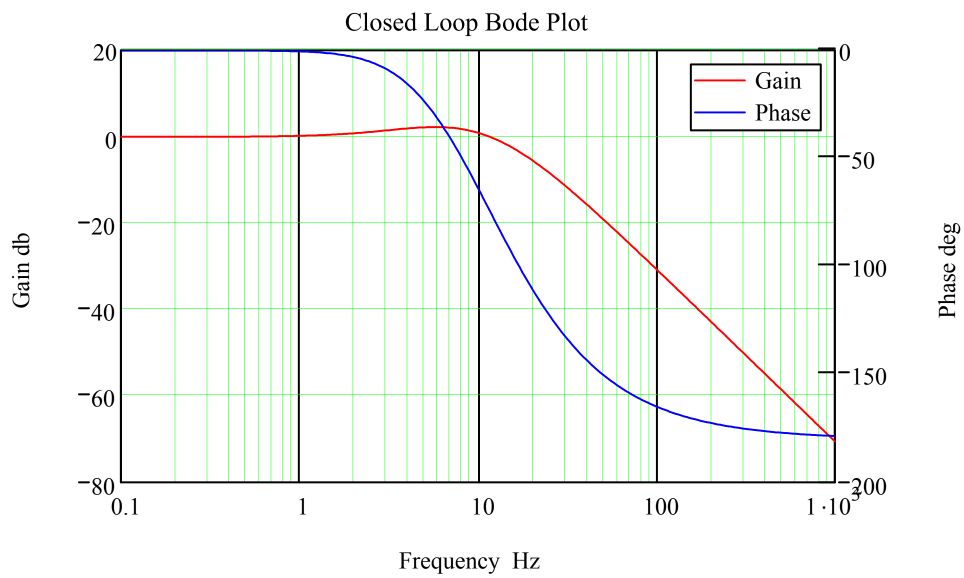
$n := 0..256$

Iterate the frequency from .01 HZ to 1000 HZ using 64 steps per decade for a total of 256 iterations.

$$hz_n := 10^{\frac{n}{64}-1}$$

$$\begin{pmatrix} M_n \\ \phi_n \end{pmatrix} := \left| \begin{array}{l} r \leftarrow \mathrm{CLTF}\left[j \cdot (2 \cdot \pi) \cdot hz_n\right] \\ a \leftarrow \dfrac{\arg(r)}{\deg} \\ a \leftarrow a - 360 \quad \text{if } n > 0 \wedge \left|a - \phi_{n-1}\right| > 100 \\ \begin{pmatrix} 20 \cdot \log(|r|) \\ a \end{pmatrix} \end{array} \right.$$

Closed loop magnitude of transfer function as a function of frequency



The increase in gain at 10 Hz is the result of the zero. If the derivative gain is in the feed back path only the gain will not rise about 0 db but the phase lag will increase.

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Graph Closed Loop Poles and Zeros

$$CLTF := \frac{\frac{K_c \cdot (s + sZero)}{s + sPole} \cdot \frac{K}{s^2}}{1 + \frac{K_c \cdot (s + sZero)}{s + sPole} \cdot \frac{K}{s^2}} \left| \begin{array}{l} expand \\ explicit \end{array} \right. \rightarrow \frac{K_c \cdot K \cdot s + K_c \cdot K \cdot sZero}{s^3 + s^2 \cdot sPole + K_c \cdot K \cdot s + K_c \cdot K \cdot sZero}$$

$$d := denom(CLTF)\ coeffs, s \rightarrow \begin{pmatrix} 8000 \cdot \pi^3 \\ 1200 \cdot \pi^2 \\ 60 \cdot \pi \\ 1 \end{pmatrix} \qquad n := numer(CLTF)\ coeffs, s \rightarrow \begin{pmatrix} 8000 \cdot \pi^3 \\ 1200 \cdot \pi^2 \end{pmatrix}$$

$$poles := polyroots(d) \qquad zeros := polyroots(n)$$

$$poles = \begin{pmatrix} -62.831854 \\ -62.831854 \\ -62.831852 \end{pmatrix} \qquad zeros = -20.943951$$