Hydraulic Cylinder Simulation

Simulating a hydraulic cylinder starts with the basic F=m*a and then is enhanced by doing a series of substitutions. The net force is equal to the force applied on the piston minus the opposing force required to push oil out the other side and the opposing force due to friction.

$$F_{net} = F_a - F_b - F_l - F_f$$

Substitute for the for the force on both sides of the piston and the frictional and load forces. Assume the friction is proportional to the velocity. Ignore static friction for now and just model the kinetic friction

$$a = \frac{P_a \cdot A_a - P_b \cdot A_b - Fl - F_f}{mass}$$

The acceleration that must be integrated to provide velocity and position.

Since a is the same as $\frac{d}{dt}v$

$$\frac{d}{dt}v = \frac{Pa \cdot Aa - Pb \cdot Ab}{mass}$$

The acceleration or rate of change in velocity

$$\frac{d}{dt}P = \frac{\beta \cdot Q}{V}$$

Expressed as a simple ODE

Since the moving piston changes the volume or ΔV on both sides of the piston the velocity of the piston must also be taken into account

$$\frac{d}{dt}P = \frac{\beta}{V}(Q + v \cdot A)$$

Calculate how pressure changes as a function of net flow

Va and Vb are the volumes of oil between the valve or pump and the cylinder piston. The subroutines Qa(Pa) and Qb(Pb) are needed to calculate the net flow.

$$\frac{d}{dt}Pa = \frac{\beta}{Va}(Q(Pa) - v \cdot Aa)$$

$$\frac{d}{dt}Pb = \frac{\beta}{Vb}(Q(Pb) + v \cdot Ab)$$

Calculate the rate of change in pressure as a function of flow and velocity.

Simulate the Hydraulic Actuator Acceleration

$P_t := 0$ Tank pressure

$Ps0 := 1500$ Initial system pressure, psia

$CylDia := 2$

$RodDia := 1.375$ sqin

$Aa := \frac{\pi}{4} \cdot CylDia^2$ $Aa = 3.142$ Square inches

$Ab := \frac{\pi}{4} \cdot \left(CylDia^2 - RodDia^2\right)$ $Ab = 1.657$ Square inches

$CylLen := 24$ inches

$W := 600$ Weight in lbf

$mass := \frac{W}{386}$ $mass = 1.554$ Slinch. A slinch is 12 slugs. 1 lb force accelerates a slinch at 1 in/s^2

$\theta := 0$ Cylinder orientation. Horizontal is 0, rod up is $\frac{\pi}{2}$ and rod down is $-\frac{\pi}{2}$

$Kf := 10$ $\frac{lbf \cdot sec}{in}$ frictional velocity constant

$Va_{min} := 3$ Dead volume. The cubic inches of oil between the valve and the piston. Try setting the volumes to 100 and then 10000.

$Vb_{min} := 3 + \frac{\pi}{4} 0.375^2 \cdot CylLen$ $Vb_{min} = 5.651$

$\beta := 160000$ The bulk modulus of oil, psi

Calculate the natural frequency of the cylinder and system

$$\omega_n := \sqrt{\frac{\beta}{mass} \cdot \left(\frac{Aa^2}{Va_{min} + Aa \cdot \frac{CylLen}{2}} + \frac{Ab^2}{Vb_{min} + Ab \cdot \frac{CylLen}{2}} \right)}$$

$\omega_n = 189.808$ Natural frequency in radians per second

$NF := \frac{\omega_n}{2 \cdot \pi}$ $NF = 30.209$ Natural frequency in Hz

$\frac{18}{\omega_n} = 0.095$ Calculate the acceleration time using Bosch's natural frequency method.

Load Force Function

Change the load force or simulate a disturbance as a function of time, angle, position or velocity. In this case add 2000 lbf or resistance and 0.4-0.4 times the move time.

$$Fl(t,\theta,x,v) := \left| \begin{array}{l} f \leftarrow 0 \\ f \leftarrow 2000\cdot\left(\dfrac{t - 0.40\cdot MoveTime}{0.03\cdot MoveTime}\right) \ \text{ if } \ 0.40\cdot MoveTime \le t < 0.43\cdot MoveTime \\ f \leftarrow 2000 \ \text{ if } \ 0.43\cdot MoveTime \le t < 0.47\cdot MoveTime \\ f \leftarrow 2000\cdot\left(\dfrac{0.5\cdot MoveTime - t}{0.03\cdot MoveTime}\right) \ \text{ if } \ 0.47\cdot MoveTime \le t < 0.5\cdot MoveTime \\ f \leftarrow f + W\cdot\sin(\theta) \end{array} \right.$$

Estimate Initial Cylinder Pressures

$Load := Fl(0,\theta,0,0)$ $\quad Load = 0$

Simulate load changes as a function of time, angle, position or velocity

$$Pa0 := \frac{0.5\cdot Ps0\cdot Ab - Load}{Aa + Ab} \qquad Pa0 = 258.951$$

$$Pb0 := \frac{0.5\cdot Ps0\cdot Aa + Load}{Aa + Ab} \qquad Pb0 = 491.049$$

The initial pressure on the A and B port should be 1/2 the supply pressure if the valve leakage from P->A and A->T is identical. The same applies to the B port

$$Pa0\cdot Aa - Pb0\cdot Ab = 0$$

Calculate the valve flow coefficient

$GPM := 26.4$

Rated valve flow at rated pressure drop.

$\Delta P := 500$

Rated pressure

$$K_v := \frac{GPM\cdot\frac{231}{60}}{\sqrt{\Delta P}} \qquad K_v = 4.545$$

Calculate the flow constant for the powered land. Yields flow in cubic inches per second

Valve Spool Response

The valve spool response can be roughly simulated by a natural frequency and a damping factor. For the purposes of this model the spool's natural frequency and damping factor are assumed to be fixed but in reality the natural frequency drops as the amplitude increases. This may be due to the fact that is takes significantly more power to move at twice the frequency.

$\omega_s := 2\cdot\pi\cdot 20 \qquad \omega_s = 125.664$

A 40 Hz valve

$\zeta_s := 0.5$

Spool damping factor.
The closer to 1 the better.

$$T_s(s) := \frac{\omega_s^{\,2}}{s^2 + 2\cdot\zeta_s\cdot\omega_s\cdot s + \omega_s^{\,2}}$$

Assume the spool is a second order system and probably under damped. The Bode plots show this to be the case

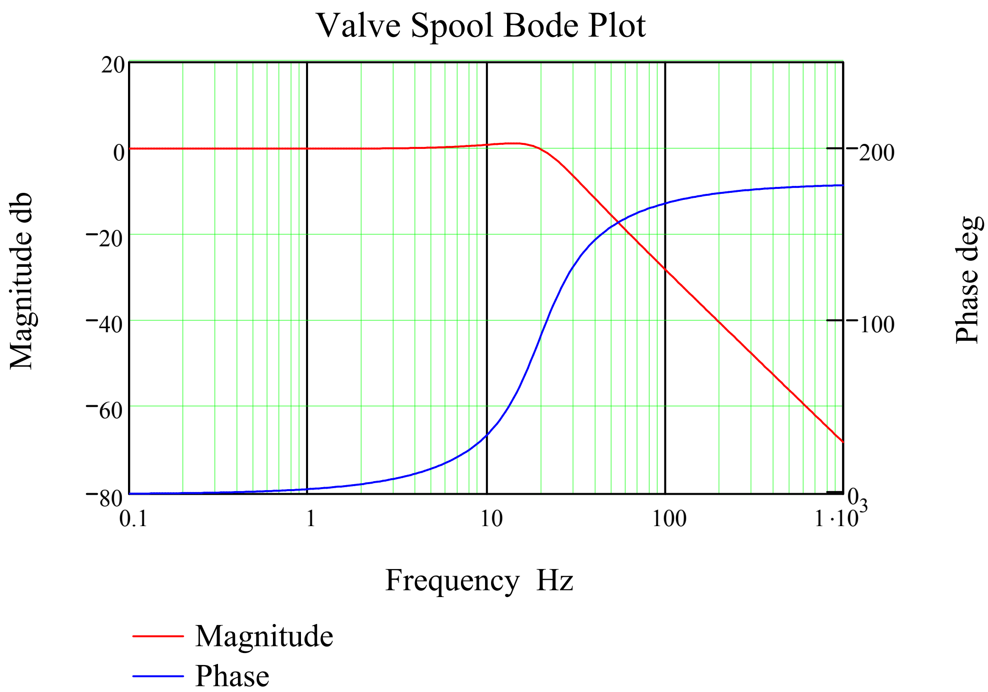
$n := 0..256$

$hz_n := 10^{\frac{n}{64}-1}$

Iterate the frequency from .01 HZ to 1000 HZ using 64 steps per decade for a total of 256 iterations.

$$\begin{pmatrix} M_n \\ \phi_n \end{pmatrix} := \left| \begin{array}{l} r \leftarrow T_s\left[j\cdot(2\cdot\pi)\cdot hz_n\right] \\ a \leftarrow \dfrac{\arg(r)}{deg} \\ a \leftarrow a - 360 \quad \text{if } n > 0 \wedge \left|a - \phi_{n-1}\right| > 100 \\ \begin{pmatrix} 20\cdot\log(|r|) \\ a \end{pmatrix} \end{array} \right.$$

Calculate magnitude and phase for T(s



The Bode plot looks similar to the plots provided by the valve manufacturers. Laplace transforms are a linear approximation to the non-linear valves but this is better than nothing. The Laplace transforms do not take into account spool travel and velocity limits.

There are two difference equations for the spool because it is a second order system. There is one for the velocity of the spool and one for the position of the spool

$$\frac{d}{dt}x_s = v_s$$ Calculate the spool position

$$\frac{d}{dt}v_s = -2\cdot\zeta_s\cdot\omega_s\cdot v_s + \omega_s^{\,2}\cdot\left(u(t) - x_s\right)$$ Calculate the spool velocity

Verifying the valve spool response

$y := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Initial spool position and velocity are 0

$u(t) := 90$ The control signal to the valve is a simple step signal of 90%

$\text{Time} := 0.1$ Simulation time

$h := 0.001$ Time between each update.

$vmax_s := 6000$ Maximum valve slew rate in %/s. The spool speed is limited. This is why the claimed frequency response is higher at lower amplitudes and spool speeds.

$$D(t,y) := \left| \begin{array}{l} \begin{pmatrix} x_s \\ v_s \end{pmatrix} \leftarrow y \\ v_a \leftarrow \omega_s^{\,2} \cdot \left(u(t) - x_s\right) - 2 \cdot \zeta_s \cdot \omega_s \cdot v_s \\ v_a \leftarrow \max\left(\min\left(v_a, \dfrac{vmax_s - v_s}{h}\right), \dfrac{-vmax_s - v_s}{h}\right) \\ v_s \leftarrow \max\left(\min\left(v_s, \dfrac{100 - x_s}{h}\right), \dfrac{-100 - x_s}{h}\right) \\ \begin{pmatrix} v_s \\ v_a \end{pmatrix} \end{array} \right.$$

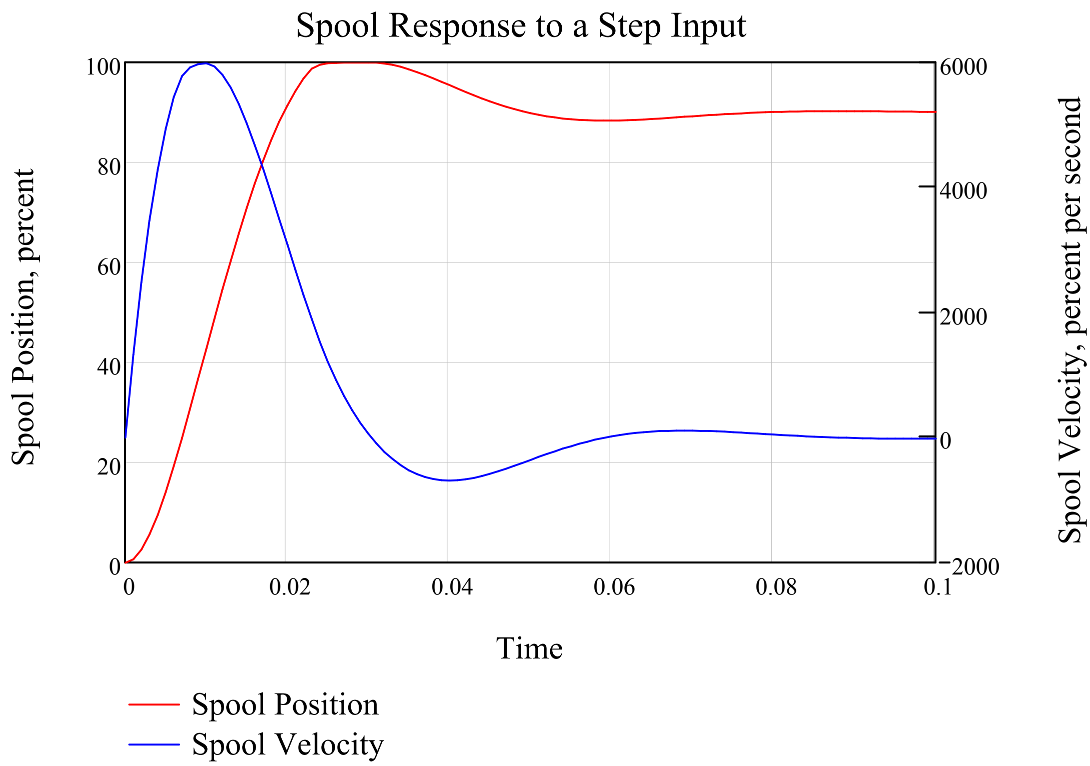
Use the differential equation to compute the spool acceleration.

limit the acceleration so the spool speed isn't exceeded.

Limit the velocity so the spool position isn't exceeeded.

Return valve spool velocity and acceleration.

$S := \text{rkfixed}(y, 0, \text{Time}, 100, D)$



One can see that the modeled valve is 97% open in about 22 milliseconds as it overshoots the 90% control signal. The over shoot occurs because the spool is a second order underdamped system. The simulations shows that the spool slew speed limit is reached at about 5 milliseconds and the spool overshoots the set point and the end of stroke at about 25 milliseconds. This is non-linear motion and the reason why Runge-Kutta should be used for non-linear simulations rather than Laplace transforums. I wish the valve manufacturers would provide this data so more accurate simulations can be made.

Estimate Valve Leakage

Calculate the leakage flow constant K_l from the leakage specification

The total leakage flow is the sum of the flow from P->A and P->B. Since the orifice is assumed to be symmetrical we can assume the flow from P->A is equal to the flow from P->B therefore the total leakage flow Ql is equal to 2 times the leakage from P->A, $Q_{pa.}$

$Q_l := 0.25 \cdot \frac{231}{60}$ $\quad Q_l = 0.963$

The specification for valve leakage for my NG10 is about .25 gpm. Convert to cubic inches per second.

The total valve leakage, Q_l, is equal to the leakage from P->A plus the leakage from P->B. Assuming the leakage is symmetrical then the total leakage is 2 times the leakage from P->A.

$$Q_l = 2 \cdot Q_{pa} = 2 \cdot K_l \cdot \sqrt{\frac{P_s}{2}}$$

The leakage flow is 2 times the flow from A->B

$$Q_l = K_l \cdot \sqrt{2 \cdot P_s}$$

Combine some constants

$$K_l := \frac{Q_l}{\sqrt{2 \cdot Ps0}}$$

Divide both sides by the square root term

$$K_l = 0.018$$

Leakage flow constant. This is very small relative to K_v.

Select valve over/under lap type.

The valve overlap is determined by x_l. The spool position, x_s, is in percent of full spool travel. $x_{l,}$ should be a small number in per cent overlap or under lap. A value of 0 signifies a 0 lapped spool whereas negative value specify under lapped spools and positive values signify overlapped spools.
$x_{.lt}$ allows one to specify a different leakage coefficient A->T and B->T that differs from P->A and P->B.

$x_l := 0$

Spool over/under lap variable in percer Set negative of under lap and positive f over lap spools. For the pressure port

$x_{lt} := x_l$

Spool over/under lap for the tank port. Normally this should be the same as the pressure port over/under lap.

Calculate the flow coefficients as a function of spool position

The flow around each edge of the spool is modeled using a simple function. Basically there are two states. Either the edge is in a normal flowing condition or it is blocked and in a leakage state. This results in two linear segments that will determine the flow coefficient as a function of the position as it goes from -100% to +100%.

$$K_{pa}(x_s) := \begin{cases} (K_v - K_l)\cdot\dfrac{x_s - x_l}{100 - x_l} + K_l & \text{if } (x_s - x_l) \geq 0 \\ K_l\cdot\left(\dfrac{100 + x_s}{100 + x_l}\right) & \text{otherwise} \end{cases}$$

Calculate the flow coefficient from the P to A ports as a function of the spool position

$$K_{at}(x_s) := \begin{cases} (K_v - K_l)\cdot\left(\dfrac{-x_s - x_{lt}}{100 - x_{lt}}\right) + K_l & \text{if } (-x_s - x_{lt}) \geq 0 \\ K_l\cdot\left(\dfrac{100 - x_s}{100 + x_{lt}}\right) & \text{otherwise} \end{cases}$$

Calculate the flow coefficient from the A to T ports as a function of the spool position

$$K_{pb}(x_s) := \begin{cases} (K_v - K_l)\cdot\dfrac{-x_s - x_l}{100 - x_l} + K_l & \text{if } x_s + x_l \leq 0 \\ K_l\cdot\left(\dfrac{100 - x_s}{100 + x_l}\right) & \text{otherwise} \end{cases}$$

Calculate the flow coefficient from the P to B ports as a function of the spool position

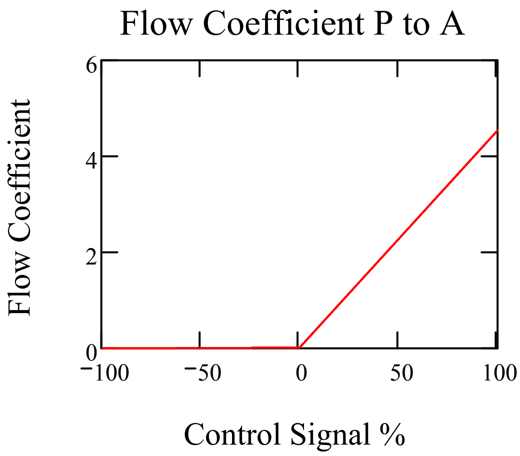
$$K_{bt}(x_s) := \begin{cases} (K_v - K_l)\cdot\left(\dfrac{x_s - x_{lt}}{100 - x_{lt}}\right) + K_l & \text{if } -x_s + x_{lt} \leq 0 \\ K_l\cdot\left(\dfrac{100 + x_s}{100 + x_{lt}}\right) & \text{otherwise} \end{cases}$$

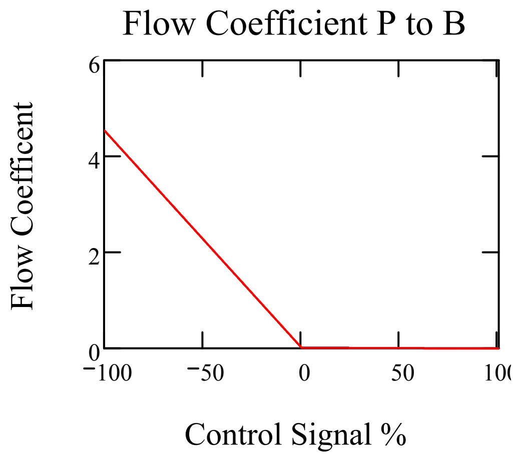
Calculate the flow coefficient from the B to T ports as a function of the spool position

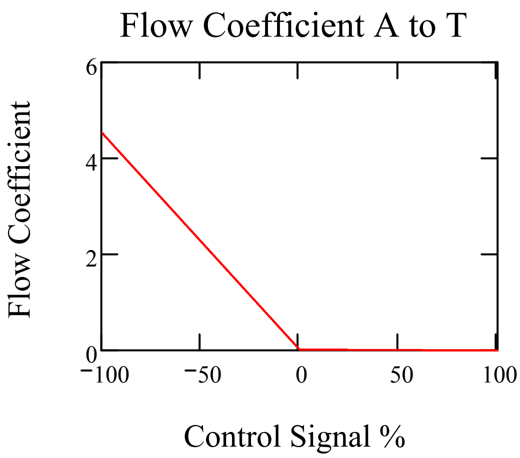
Note, the spools are assume to be cut so that the flow changes linearly with spool position. This is not always the case. There are 'curvilinear' and 'nick' spools. Modeling either one of these spools would require modifications to the four flow coefficient functions above. For instance, and 'nick' spool can be simulated by adding a third linear segment with a higher gain when the spool reaches 20% or 40% depending on the spool.

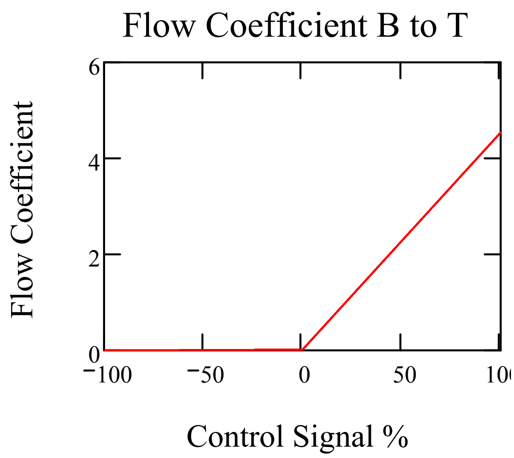
Sanity check the flow constant formula by graphing the flow coefficients as a function of the control signal. Note that the control signal goes from -1 to 1.

$x := -100, -099.9 .. 100$









Pressure Gain

Plot the A and B port pressures assuming non-linear leakage.
When testing pressure gain oil flows into the P port. The A port is capped or dead headed. Some oil will still leak through the valve to the tank. The B port is tested in a similar way but the B port pressure goes up when the control signal is negative. The pressure gain is only important when controlling pressure in static situations. During dynamic testing the equation

$\frac{d}{dt}P = \frac{\beta\cdot(Q(t) - v\cdot A)}{V(t)}$ is more more important.

When controlling force the pressure gain is more important than the individual pressures. When using a double rodded cylinder the force is roughly

$$\text{Force} = \text{PistonArea}\cdot\left[P_a(x_s) - P_b(x_s)\right]$$

However, a single rodded cylinder is used in most applications so the formula becomes

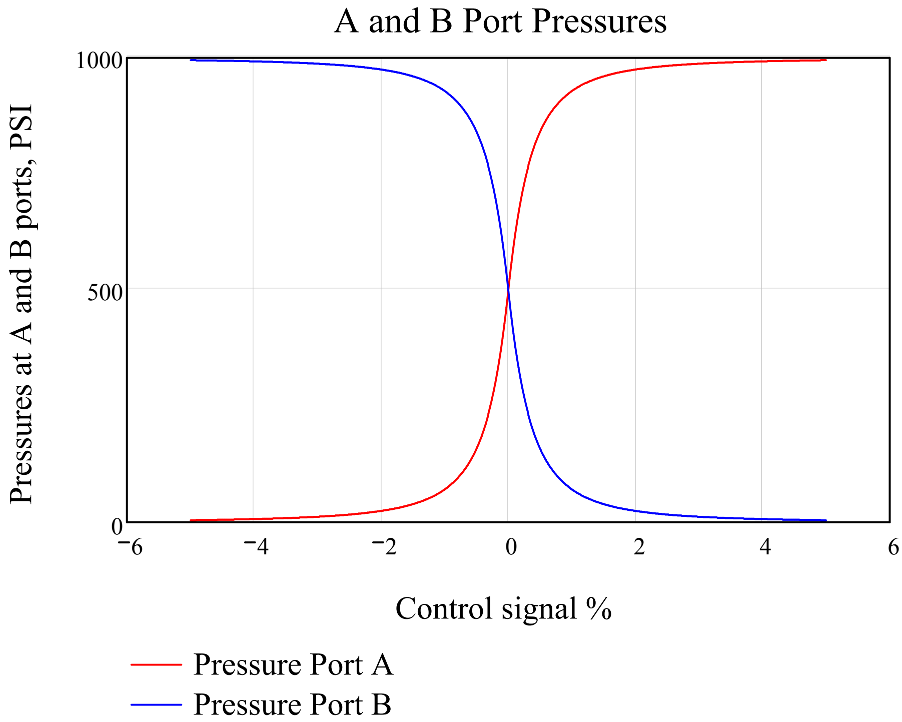
$$\text{Force} = \left[\text{Area}_a\cdot P_a(x_s) - \text{Area}_b\cdot P_b(x_s)\right]$$

Again, these formulas apply to static force control applications. Dynamic force control is more complicated.

$\Delta P := 1000$ psi roughly 70 bar

$$P_a(x_s) := \frac{P_t\cdot K_{at}(x_s)^2 + K_{pa}(x_s)^2\cdot\Delta P}{K_{pa}(x_s)^2 + K_{at}(x_s)^2} \qquad P_b(x_s) := \frac{P_t\cdot K_{bt}(x_s)^2 + K_{pb}(x_s)^2\cdot\Delta P}{K_{pb}(x_s)^2 + K_{bt}(x_s)^2}$$

$x := -5, -4.99 .. 5$



Simulating Pump and Accumulators

A real hydraulic system does not have a constant supply pressure. The supply pressure drops when a lot of flow is required. To offset this accumulators are used to supply energy. This allow the designer to use a pump that supplies just a little more than the average flow. During slow or dwell times the oil/energy is stored in accumulators and during times when the flow exceeds the pump capacity the accumulator can make up the difference.

$GPM := 10$ $\quad GPM \cdot \frac{231}{60} = 38.5$ Pump capacity in in^3/sec

$PPB := 200$ Pump proportional band, psi

$\tau p := 0.050$ Pump time constant in seconds

$AccumSize := 5$ Accumulator size in gallons

$V_a := AccumSize \cdot \frac{gal}{in^3}$ $\quad V_a = 1155$ Accumulator size in cubic inches

$\gamma := 1.4$ Ratio of specific heat Cp/Cv

Calculate the Supply Pressure and Accumulator Volume As A Function of Tim

$P_0 := 0.9 \cdot Ps0$ $\quad P_0 = 1350$ Pre charge pressure

$K := P_0 \cdot V_a^{\gamma}$ $\quad K = 2.618 \times 10^7$ Calculate K, nrT, using pre charge values calculated above for pressure calculations below.

$K = Ps \cdot V_g^{\gamma} \;\Big|\; \begin{array}{l} \text{solve}, V_g \\ \text{explicit} \end{array} \rightarrow e^{\frac{\ln\left(\frac{K}{Ps}\right)}{\gamma}}$ Solve for the gas volume at the initial system pressure

$Vg0 := e^{\frac{\ln\left(\frac{K}{Ps0}\right)}{\gamma}}$ $\quad Vg0 = 1071.268$ Initial gas volume in cubic inches

$Ps(Vg) := \frac{K}{Vg^{\gamma}}$ $\quad Ps(Vg0) = 1500$ Supply pressure as a function of the accumulator volume.

Total Flow Equations

Qa and Qb are the net flow to either side of the cylinder piston. The total flow equations are important since they determine the rate of change in pressure,$\Delta P=\beta^*\Delta V/V$ or $\Delta P=\beta^*Q^*\Delta t$. The current flow equation has four parts. The top equation calculates the flow from the supply to the A or B port and the second equation calculates the flow from the A or B port to the tank.The leakage terms are very important. Without the leakage terms the system will tend to oscillate for an unrealistically long time since there is little if any energy loss when the valve is shut. When accelerating there is plenty of energy loss due to the pressure drop across the valve. You can see there isn't any hint of oscillating because the damping factor is very high when the spools is wide open.

$$Q_{pa}\left(Pa, x_s, Vg\right) := K_{pa}\left(x_s \cdot 100\right) \cdot \mathrm{sign}(Ps(Vg) - Pa) \cdot \sqrt{\left|Ps(Vg) - Pa\right|}$$

$$Q_{at}\left(Pa, x_s\right) := -K_{at}\left(x_s \cdot 100\right) \cdot \mathrm{sign}\left(Pa - P_t\right) \cdot \sqrt{\left|Pa - P_t\right|}$$

$$Q_{pb}\left(Pb, x_s, Vg\right) := K_{pb}\left(x_s \cdot 100\right) \cdot \mathrm{sign}(Ps(Vg) - Pb) \cdot \sqrt{\left|Ps(Vg) - Pb\right|}$$

$$Q_{bt}\left(Pb, x_s\right) := -K_{bt}\left(x_s \cdot 100\right) \cdot \mathrm{sign}\left(Pb - P_t\right) \cdot \sqrt{\left|Pb - P_t\right|}$$

$$Qa\left(Pa, x_s, Vg\right) := Q_{pa}\left(Pa, x_s, Vg\right) + Q_{at}\left(Pa, x_s\right)$$

$$Qb\left(Pb, x_s, Vg\right) := Q_{pb}\left(Pb, x_s, Vg\right) + Q_{bt}\left(Pb, x_s\right)$$

Target Generator for Position, Velocity and Acceleration

Generation motion profile using a cosine ramp target generator. This target generator will calculate a position, velocity and acceleration at time t. The motion profile assumes the ramp up, constant velocity and ramp down sections each take 1/3 of the total move time. All that is required is the move time and the distance to move.

$x0 := 4$ Initial position

$\text{MoveTime} \equiv 1$

$\text{MoveDist} \equiv 16$

$\text{CycleTime} := 3 \cdot \text{MoveTime} \qquad \text{CycleTime} = 3$

$$\text{MaxVel} := \frac{3}{2} \cdot \frac{\text{MoveDist}}{\text{MoveTime}} \qquad \text{MaxVel} = 24$$

$$\text{AvgAcc} := \frac{9}{2} \cdot \frac{\text{MoveDist}}{\text{MoveTime}^2} \qquad \text{AvgAcc} = 72$$

$$\frac{\text{MoveDist} \cdot (\text{Aa} + \text{Ab})}{2 \cdot \text{MoveTime}} = 38.386$$

Average flow required

Calculate the feed forwards

When extending against a load

$\rho v := 1$ — The spool is symmetrical

$Ape := Aa$ — $Ape = 3.142$

$FL := Fl(0,0,0,0)$ — $FL = 0$

$\rho c := \frac{Aa}{Ab}$ — $\rho c = 1.896$ — Ratio of the powered end area the exhausting end area

Compute maximum steady state velocity in each direction to estimate velocity feed forwards

$$vss\_ext := K_V \cdot \sqrt{\frac{Ps0 \cdot Ape - FL}{Ape^3 \cdot \left(1 + \frac{\rho v^2}{\rho c^3}\right)}}$$

$vss\_ext = 52.331$

Calculate the maximum velocity, in inches per second, when the control output is one using the VCCM equation.
vss_ext is the velocity stead state while extending.

When retracting

$Ape := Ab$

$FL := Fl(0,0,0,0)$ — $FL = 0$ — No load when retracting

$\rho c := \frac{Ab}{Aa}$ — $\rho c = 0.527$ — Ratio of the powered end area the exhausting end area

$$vss\_ret := K_V \cdot \sqrt{\frac{Ps0 \cdot Ape - FL}{Ape^3 \cdot \left(1 + \frac{\rho v^2}{\rho c^3}\right)}}$$

$vss\_ret = 38.002$

velocity steady state while retracting in inches per second

The Simulation Using Runge-Kutta.

Execute the equations simultaneously by putting the equations and results in a matrix.
There are now eight differential equations that are computed simultaneously.

$$\mathrm{ActAcc}(t, Pa, Pb, x, v, xt, vt) := \frac{(Pa \cdot Aa - Pb \cdot Ab) - Kf \cdot v - Fl(t, \theta, x, v)}{mass}$$

Calculate the actual acceleration

$$y := \begin{pmatrix} 0 \\ 0 \\ Pa0 \\ Pb0 \\ 0 \\ x0 \\ 0 \\ Qp \leftarrow 0 \\ Vg0 \end{pmatrix}$$

Initial spool position
initial spool velocity
Initial A port pressure
Initial B port pressure
Initial velocity
Initial position
Initial control output
Initial accumulator gas volume

y is the current state

$$D(t,y) := \left| \begin{array}{l} \begin{pmatrix} xt \\ vt \\ at \end{pmatrix} \leftarrow r(t, MoveTime, MoveDist) \\ \begin{pmatrix} x_s \\ v_s \\ Pa \\ Pb \\ v \\ x \\ u \\ Qp \\ Vg \end{pmatrix} \leftarrow y \\ a \leftarrow ActAcc(t, Pa, Pb, x, v, xt, vt) \\ \begin{pmatrix} u \\ u' \end{pmatrix} \leftarrow PID(u, xt, vt, at, x, v, a) \\ \left[\begin{array}{c} v_s \\ \left| \begin{array}{l} v_a \leftarrow \omega_s^{\,2} \cdot (u - x_s) - 2 \cdot \zeta_s \cdot \omega_s \cdot v_s \\ v_a \leftarrow \max\left(\min\left(v_a, \dfrac{vmax_s - v_s}{\Delta t}\right), \dfrac{-vmax_s - v_s}{\Delta t}\right) \end{array} \right. \\ \dfrac{\beta}{Va_{min} + x \cdot Aa} \cdot \left(Qa(Pa, x_s, Vg) - v \cdot Aa\right) \\ \dfrac{\beta}{Vb_{min} + (CylLen - x) \cdot Ab} \cdot \left(Qb(Pb, x_s, Vg) + v \cdot Ab\right) \\ a \\ v \\ u' \\ \dfrac{1}{\tau p} \cdot \left(\max\left(\min\left(\dfrac{Ps0 - Ps(Vg)}{PPB}, 1 \right), 0 \right) \cdot GPM \cdot \dfrac{231}{60} - Qp \right) \\ Q_{pa}(Pa, x_s, Vg) + Q_{pb}(Pb, x_s, Vg) - Qp \end{array} \right] \end{array} \right.$$

Calculate the target position, velocity and acceleration.

Assign the current state to local variables for easier reading and debuging

Calculate the acceleration

Calculate the control outpu

integrate spool velocity to get spool position. Integrate spool acceleration to get spool velocity
Integrate the rate of pressure change to get pressure on the cap side and on the rod side

integrate actuator acceleration to get actuator velocity and integrate cylinder velocity to get cylinder position.
Integrate the rate of change in pump flow to get the current pump flow.
Integrate the change in flow in or out of the accumulator to calculate the gas volume of the accumulator.

$SimTime := 5 \cdot MoveTime + 1$ Simulation time

$N := \dfrac{SimTime}{\Delta t}$ $\quad N = 30000$ Number of iterations

$S := rkfixed(y, 0, SimTime, N, D)$

$t := S^{\langle 0 \rangle}$ $x_s := S^{\langle 1 \rangle}$ $Vg := S^{\langle 3 \rangle}$ $Pa := S^{\langle 3 \rangle}$ $Pb := S^{\langle 4 \rangle}$

$v := S^{\langle 5 \rangle}$ $x := S^{\langle 6 \rangle}$ $u := S^{\langle 7 \rangle}$ $Qp := S^{\langle 8 \rangle}$ $Vg := S^{\langle 9 \rangle}$

$n := 0 .. N$

$$\begin{pmatrix} xt_n \\ vt_n \\ at_n \end{pmatrix} := r(n \cdot \Delta t, MoveTime, MoveDist)$$

$a_n := ActAcc(t_n, Pa_n, Pb_n, x_n, v_n, xt_n, vt_n)$

$CO_n := PID(u_n, xt_n, vt_n, at_n, x_n, v_n, a_n)_0$

$DifFrc_n := (Pa_n \cdot Aa - Pb_n \cdot Ab)$

$NetFrc_n := (Pa_n \cdot Aa - Pb_n \cdot Ab) - Kf \cdot v_n$

$Fric_n := Kf \cdot v_n$

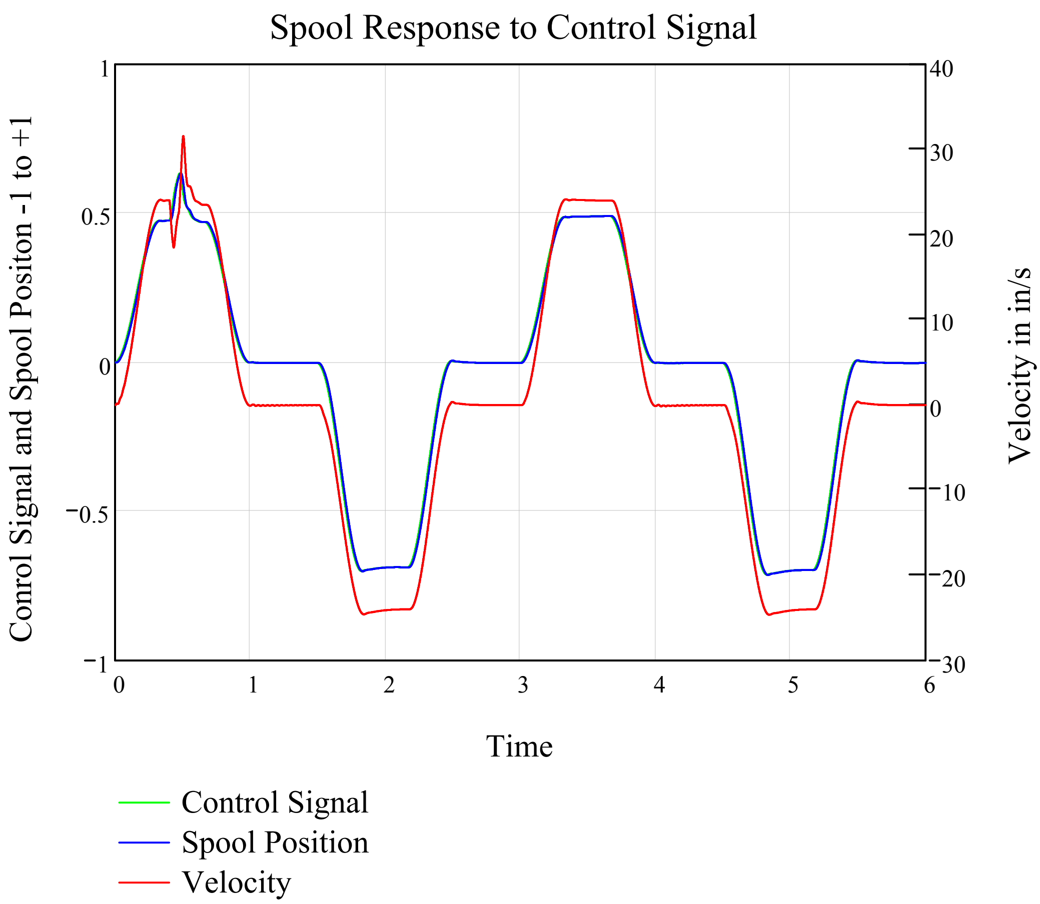
$uff_n := Kv\_ext \cdot vt_n + Ka \cdot at_n$

Hydraulic Cylinder Simulation

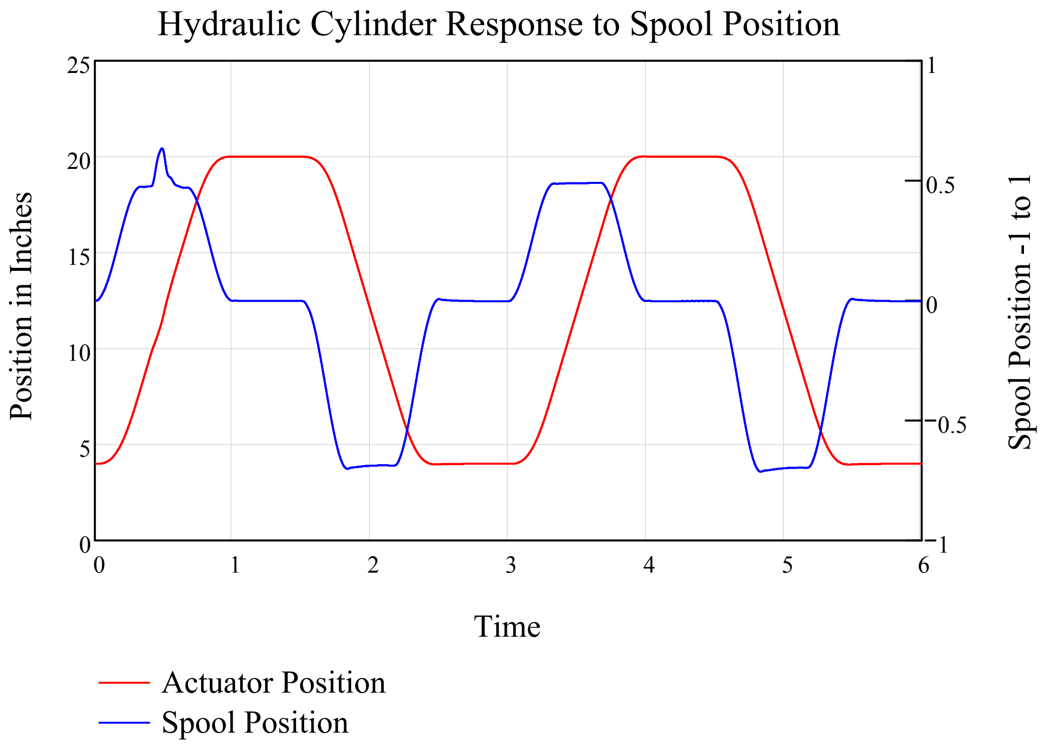
	t	xs	vs	Pa	Pb	v	x	u	Qp
	0	1	2	3	4	5	6	7	8
0	0	0	0	258.951	491.049	0	4	0	0
1	0	$3.932\cdot10^{-9}$	0	259.642	491.188	0	4	$3.988\cdot10^{-7}$	$1.774\cdot10^{-7}$
2	0	$3.125\cdot10^{-8}$	0	260.33	491.327	0	4	$1.606\cdot10^{-6}$	$7.088\cdot10^{-7}$
3	0.001	$1.048\cdot10^{-7}$	0.001	261.013	491.468	0.001	4	$3.637\cdot10^{-6}$	$1.593\cdot10^{-6}$
4	0.001	$2.468\cdot10^{-7}$	0.001	261.691	491.609	0.002	4	$6.502\cdot10^{-6}$	$2.827\cdot10^{-6}$
5	0.001	$4.789\cdot10^{-7}$	0.001	262.361	491.751	0.003	4	-0	$4.412\cdot10^{-6}$
6	0.001	$8.22\cdot10^{-7}$	0.002	263.022	491.895	0.004	4	-0	$6.344\cdot10^{-6}$
7	0.001	$1.297\cdot10^{-6}$	0.003	263.673	492.041	0.006	4	-0	$8.623\cdot10^{-6}$
8	0.002	$1.922\cdot10^{-6}$	0.004	264.312	492.188	0.008	4	-0	0
9	0.002	$2.719\cdot10^{-6}$	0.004	264.938	492.339	0.01	4	-0	0
10	0.002	$3.704\cdot10^{-6}$	0.005	265.549	492.492	0.012	4	-0	0
11	0.002	$4.896\cdot10^{-6}$	0.007	266.145	492.648	0.015	4	-0	0
12	0.002	$6.312\cdot10^{-6}$	0.008	266.724	492.808	0.017	4	-0	0
13	0.003	$7.97\cdot10^{-6}$	0.009	267.285	492.971	0.02	4	-0	0
14	0.003	$9.885\cdot10^{-6}$	0.01	267.826	493.138	0.023	4	-0	0
15	0.003	0	0.012	268.347	493.308	0.026	4	-0	0
16	0.003	0	0.013	268.847	493.483	0.03	4	-0	0
17	0.003	0	0.015	269.325	493.663	0.033	4	-0	0
18	0.004	0	0.016	269.779	493.847	0.037	4	-0	0
19	0.004	0	0.018	270.21	494.035	0.041	4	-0	0
S = 20	0.004	0	0.02	270.617	494.229	0.044	4	-0	0
21	0.004	0	0.022	270.998	494.427	0.049	4	-0	0
22	0.004	0	0.023	271.354	494.631	0.053	4	-0	0
23	0.005	0	0.025	271.684	494.84	0.057	4	-0	0
24	0.005	0	0.027	271.987	495.054	0.061	4	-0	0
25	0.005	0	0.029	272.264	495.273	0.066	4	-0	0
26	0.005	0	0.032	272.514	495.498	0.07	4	-0	0
27	0.005	0	0.034	272.737	495.728	0.074	4	-0	0
28	0.006	0	0.036	272.932	495.964	0.079	4	-0	0
29	0.006	0	0.038	273.101	496.205	0.083	4	-0	0
30	0.006	0	0.041	273.243	496.452	0.088	4	-0	0
31	0.006	0	0.043	273.358	496.704	0.092	4	-0	0
32	0.006	0	0.045	273.447	496.961	0.097	4	-0	0
33	0.007	0	0.048	273.51	497.223	0.101	4	-0	0
34	0.007	0	0.05	273.547	497.491	0.106	4	-0	0
35	0.007	0	0.053	273.559	497.763	0.11	4	-0	0
36	0.007	0	0.056	273.546	498.04	0.114	4	-0	0
37	0.007	0	0.058	273.51	498.322	0.119	4	-0	0
38	0.008	0	0.061	273.451	498.609	0.123	4	-0.001	0
39	0.008	0	0.064	273.369	498.9	0.127	4	-0.001	0
40	0.008	0	0.067	273.266	499.195	0.131	4	-0.001	0
41	0.008	0	0.069	273.142	499.495	0.135	4	-0.001	0

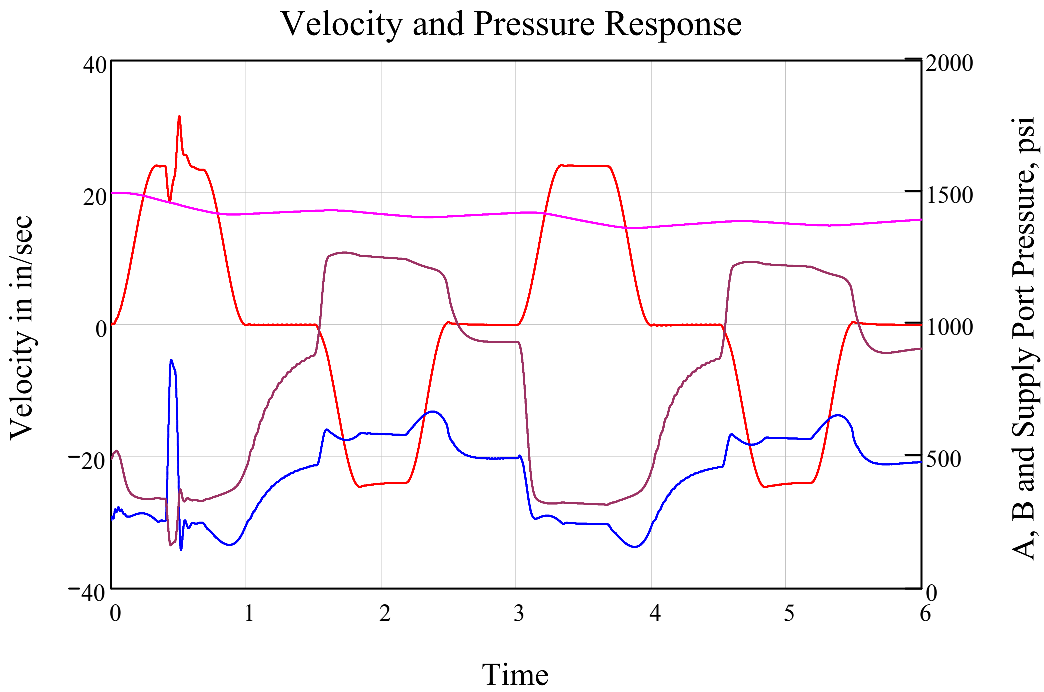
Hydraulic Cylinder Simulation

The first Spool Response graph shows the control signal, valve spool position and as a function of time. The disturbance starting at about 0.4 seconds is caused by the load force function $Fl(t, \theta, x, v)$. The load force function is handy for simulating how the hydraulic system and controller will respond to varying loads and conditions.



Hydraulic Cylinder Simulation



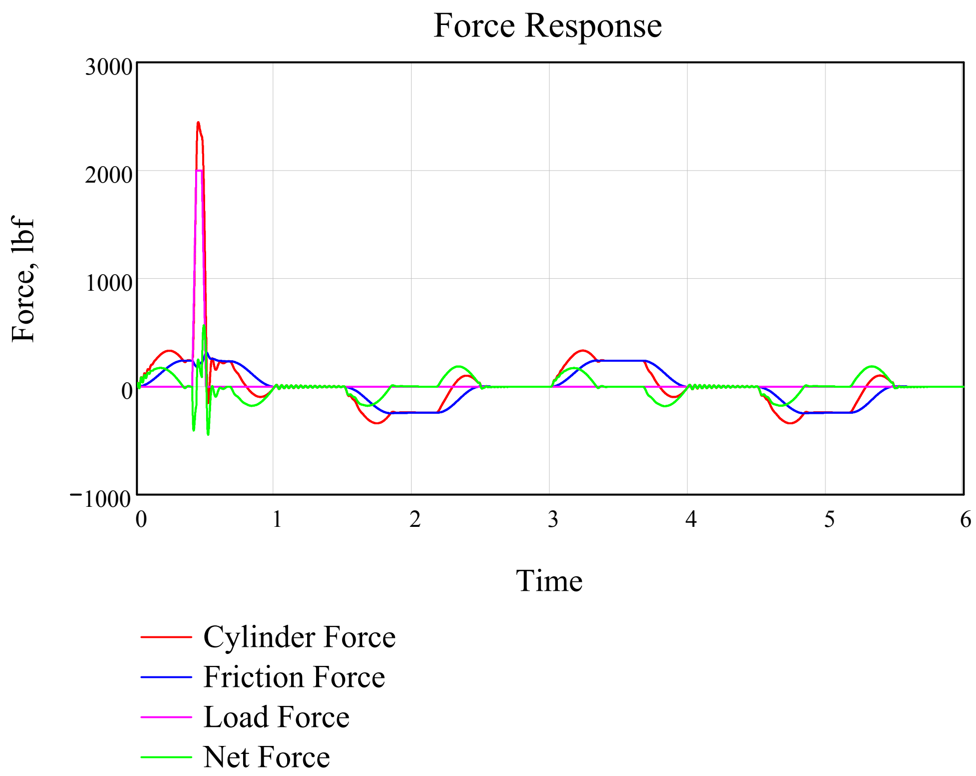


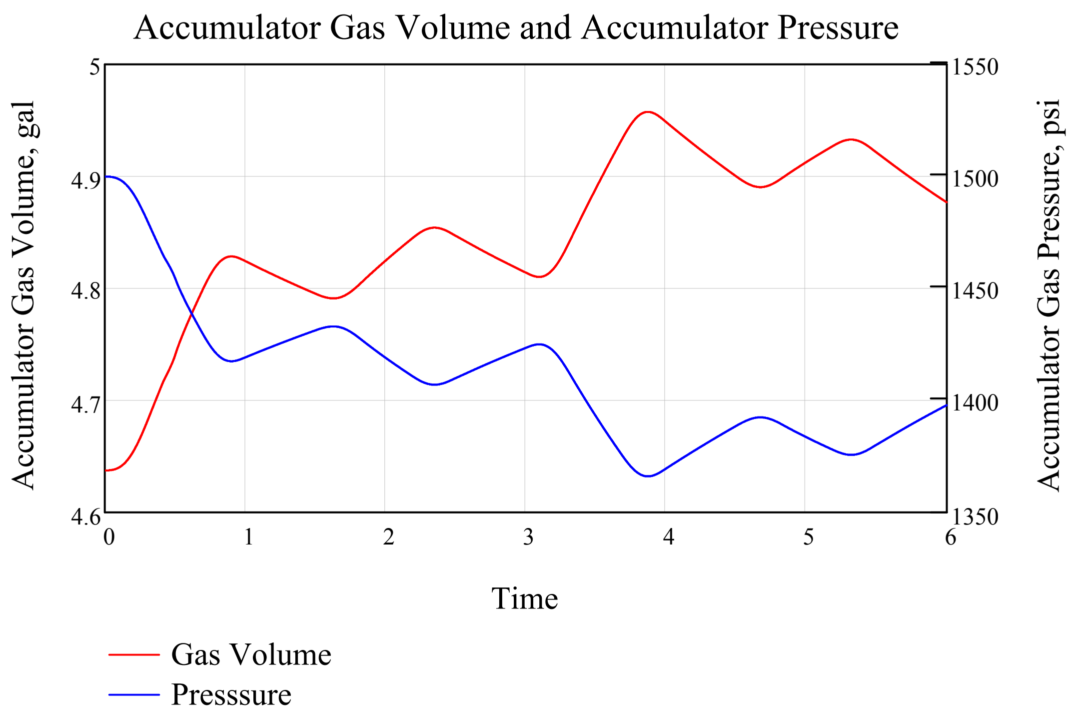
The load can induce a large pressure difference across the piston.
The supply pressure is plotted to compare with the A and B port pressure. When the valve shut quickly pressure spikes are induced and these pressure spikes can cause the oil to flow back into the accumulator. This is why the code for flow must use

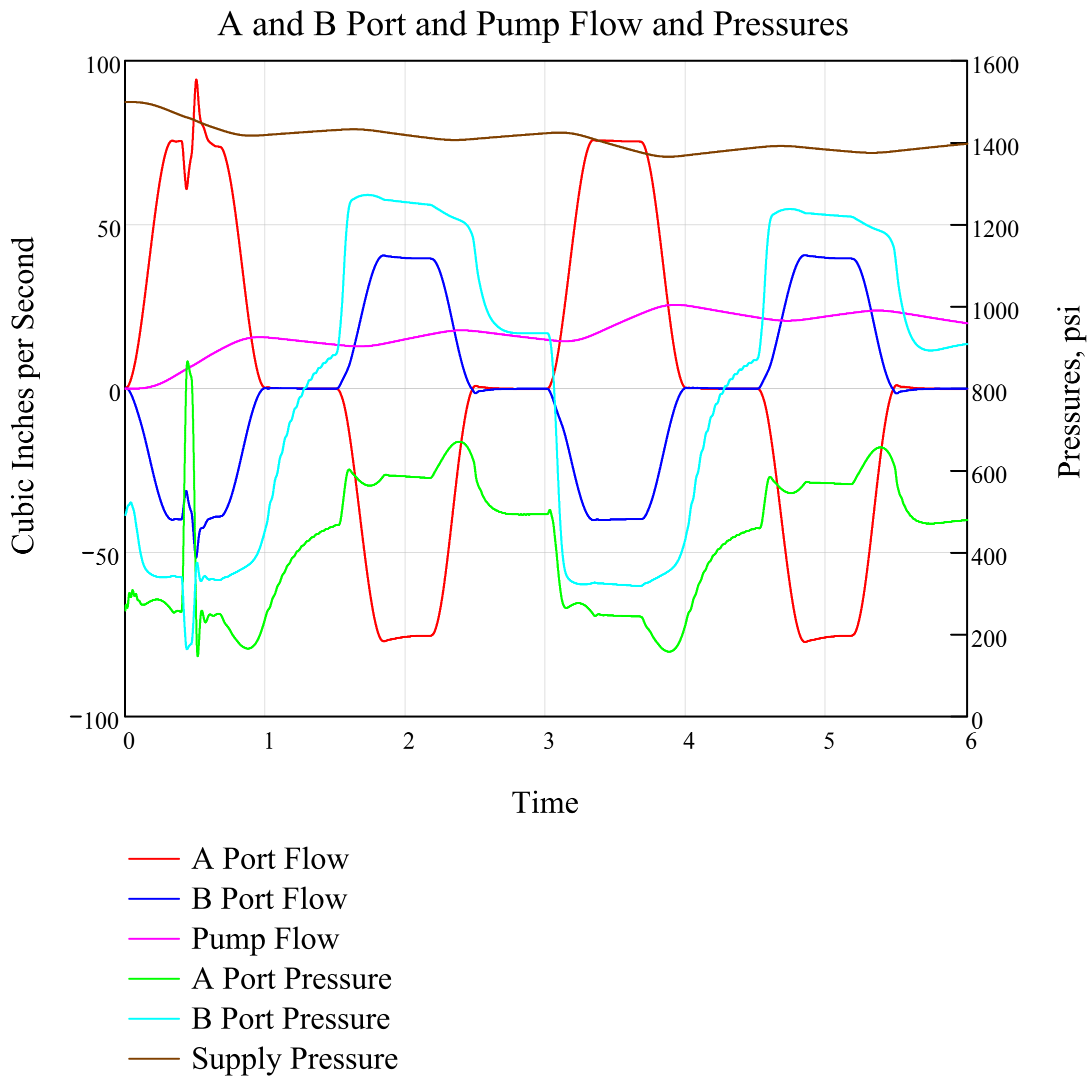
$$\mathrm{sign}(\mathrm{Ps}(\mathrm{Vg}) - \mathrm{Pa})\cdot\sqrt{|\mathrm{Ps}(\mathrm{Vg}) - \mathrm{Pa}|}$$

when computing the flow. This avoids errors caused by taking the square root of a negative number.

Hydraulic Cylinder Simulation



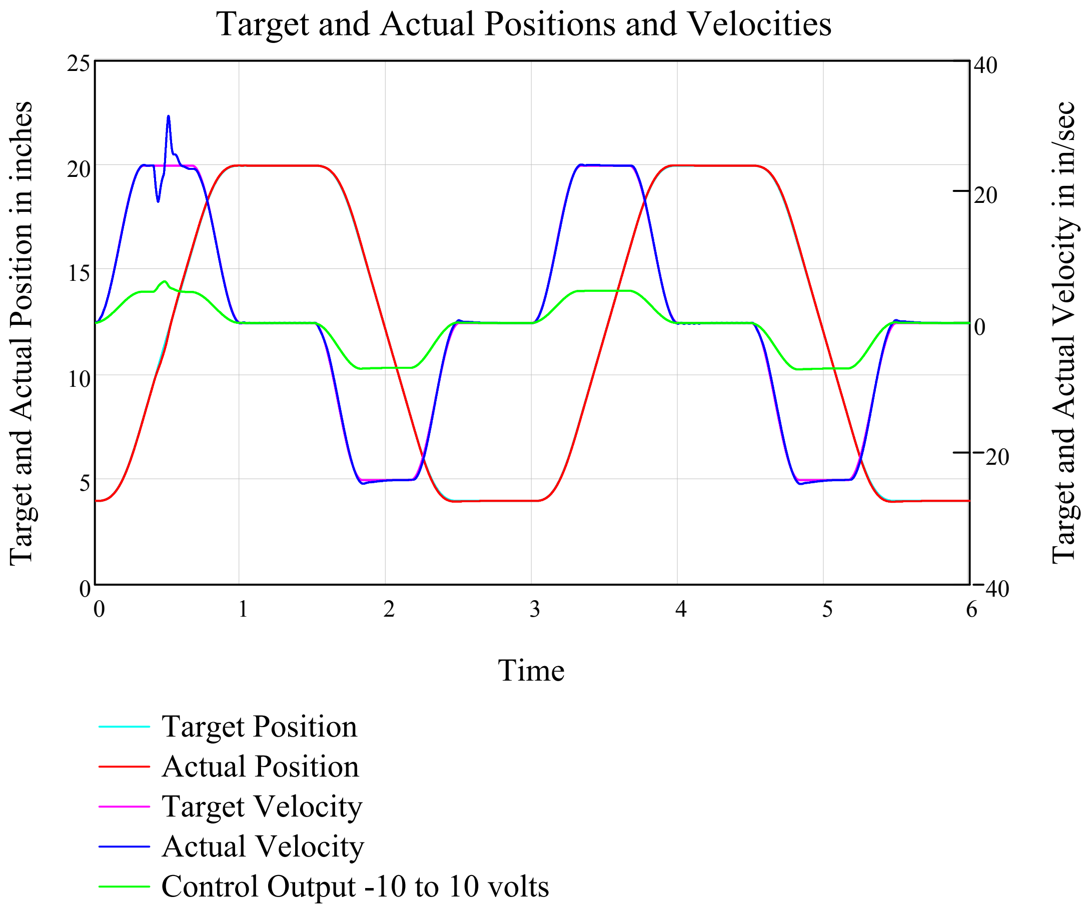




The pump flow is not as great as the flow into the cylinders A port. This can happen because the extra oil comes from the accumulator. Accumulators are important keeping the system pressure as constant as possible since hydraulic pumps respond too slowly in dynamic systems.

Closed Loop Control

The controller reacts to the disturbance quickly. The actual position follows the target position almost perfectly except at the disturbance at 0.4 seconds.



Gains are calculated using pole placement.

$Ki = 3.96544$	Integrator gain
$Kp = 0.522$	Proportional gain
$Kd = 0.004$	Derivative gain
$Kv\_ext = 0.019$	Velocity feed forward extend
$Kv\_ret = 0.026$	Velocity feed forward retract
$Ka = 0.00018$	Acceleration feed forward

$$\sum_{n} \frac{\left(r(n \cdot \Delta t, MoveTime, MoveDist)_0 - x_n\right)^2}{N} = 0.001202$$

Normalized sum of squared errors. Adjust gains to minimize

$$\sum_{n} \left(u_n\right)^2 = 35.012$$

Minimize the PID control output number by setting the feed forwards correctly