Accumulator Calculations for Point to Point Motion Profile

Accumulator Sizing

$CylDia := 3.25$		inches
$RodDia := 1.75$		inches
$Stroke := 2.5$		Stroke in inches
$t_{ext} := 0.3$		Time to extend
$vel_{ext} := \frac{1.5 \cdot Stroke}{t_{ext}}$	$vel_{ext} = 12.5$	Extend velocity
$acc_{ext} := \frac{4.5 \cdot Stroke}{t_{ext}^{2}}$	$acc_{ext} = 125$	Extend acceleration
$t_{ret} := 0.3$		Time to retract
$vel_{ret} := \frac{1.5 \cdot Stroke}{t_{ret}}$	$vel_{ret} = 12.5$	Retract velocity
$acc_{ret} := \frac{4.5 \cdot Stroke}{t_{ret}^{2}}$	$acc_{ret} = 125$	Retract acceleration
$P_s := 1500$		Steady state pressure. psi Pump pressure setting.
$P_{min} := 1400$		Minimum pressure. psi
$\gamma := 1.4$		Ratio of specific heat Cp/Cv
$Area_{cap} := \frac{\pi}{4} \cdot CylDia^{2}$	$Area_{cap} = 8.296$	sq in
$Area_{rod} := \frac{\pi}{4} \cdot \left(CylDia^{2} - RodDia^{2}\right)$	$Area_{rod} = 5.89$	Rod side area
$vol_{ext} := Area_{cap} \cdot Stroke$	$vol_{ext} = 20.739$	Volume to extend
$vol_{ret} := Area_{rod} \cdot Stroke$	$vol_{ret} = 14.726$	Volume to retract
$vol_{ext} + vol_{ret} = 35.466$		Volume per cycle

Valve Sizing

$PeakFlow := \max\left(vel_{ext} \cdot Area_{cap}, vel_{ret} \cdot Area_{rod}\right) \cdot \frac{60}{231}$	$PeakFlow = 26.934$	GPM
$Valve_{gpm} := 25$		Round up to next valve size. Assume it is 15 GPM

$CyclesPerMinute := \frac{60}{t_{ext} + t_{ret}}$ $\qquad CyclesPerMinute = 100$

$Qleak := .1 \cdot \frac{231}{60}$ $\qquad Qleak = 0.385$ $\qquad$ Leakage in cu in/sec/sec

$ReqGPM := \frac{\left(vol_{ext} + vol_{ret}\right) \cdot CyclesPerMinute + Qleak}{231}$ $\qquad$ Required GPM

$ReqGPM = 15.355$

The pump GPM may need to be much bigger to keep the cycle time less than the maximum cycle time.

$GPM := 20$ $\qquad$ From the pump spec sheets.

Calculate the average power required for the traverse. Note Mathcad does the unit conversion and an efficiency of 0.95 is assumed.

$HP_{traverse} := \frac{ReqGPM \cdot \frac{gal}{min} \cdot P_s \cdot \frac{lbf}{in^2}}{0.95}$ $\qquad HP_{traverse} = 14.143\, hp$

$HP := 20$ $\qquad$ From the pump spec sheets

The pump proportional band is the difference in pressure from between being a 0% stock and 100% stroke. This number should come from the pump spec sheets. The PPB affects how much bigger the pump must be to keep the pressure in the desired operating band between P1 and P.min. This is due to the fact that the pump isn't at full flow while the pressure is in this band. The smaller the PPB the less the pump must be oversized.

$PPB := 200$ $\qquad$ psi

Pump flow in cubic inches per second. An error occurs if the application requires more power than available from the pump.

$$Qp(P) := \left| \begin{array}{l} Q \leftarrow \min\left(1, \frac{P_s - P}{PPB}\right) \cdot GPM \cdot \frac{231}{60} \quad \text{if } HP \ge \frac{GPM \cdot P}{1714 \cdot .95} \\ \text{otherwise} \\ \quad \left| \begin{array}{l} \text{error("not enough power ")} \\ Q \leftarrow 0 \end{array} \right. \\ Q \end{array} \right.$$

$Qmax := GPM \cdot \frac{231}{60}$ $\qquad Qmax = 77$ $\qquad \frac{cuin}{sec}$

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Set extend, retract and dwell times.

$t_{extdwell} := 0.0$

$t_{retdwell} := 0.0$

Dwell between extend and retract moves

$Tdwell1 := 0.0$

$t_0 := 0$	$t_0 = 0$	Start of the cycle
$t_1 := t_0 + \frac{t_{ext}}{3}$	$t_1 = 0.1$	Time at end of extend acceleration ramp
$t_2 := t_1 + \frac{t_{ext}}{3}$	$t_2 = 0.2$	Time at the start of the deceleration ramp.
$t_3 := t_2 + \frac{t_{ext}}{3}$	$t_3 = 0.3$	Time to get to the extend position
$t_4 := t_3 + t_{extdwell}$	$t_4 = 0.3$	Time at the end of the dwell time
$t_5 := t_4 + \frac{t_{ret}}{3}$	$t_5 = 0.4$	Time at the end of the retract acceleration ramp
$t_6 := t_5 + \frac{t_{ret}}{3}$	$t_6 = 0.5$	Time at the start of the retract deceleration ramp.
$t_7 := t_6 + \frac{t_{ret}}{3}$	$t_7 = 0.6$	Time when back at the start point
$t_8 := t_7 + t_{retdwell}$	$t_8 = 0.6$	Total cycle time.

Calculate flows based on desired velocity profile during cycle. Note, actual velocity will vary from specifications. It may be necessary to have a Q2 or Q3 for two or three axes on the same system.

$$Q1(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, t_8) \\ Area_{cap} \cdot acc_{ext} \cdot (t - t_0) \ \text{ if } t_0 \le t < t_1 \\ Area_{cap} \cdot vel_{ext} \ \text{ if } t_1 \le t < t_2 \\ Area_{cap} \cdot \left[vel_{ext} - acc_{ext} \cdot (t - t_2) \right] \ \text{ if } t_2 \le t < t_3 \\ Area_{rod} \cdot acc_{ret} \cdot (t - t_4) \ \text{ if } t_4 \le t < t_5 \\ Area_{rod} \cdot vel_{ret} \ \text{ if } t_5 \le t < t_6 \\ Area_{rod} \cdot \left[vel_{ret} - acc_{ret} \cdot (t - t_6) \right] \ \text{ if } t_6 \le t < t_7 \\ 0 \ \text{ otherwise} \end{array} \right.$$

$\Delta t := .01$ simulation time increment

Compute the change in gas volume in the accumulator during motion. This value is complement of the change in oil volume.

$\Delta Vol_0 := 0$ Initialize ΔVol

$N := \frac{6 \cdot t_8}{\Delta t}$ $N = 360$ Compute the number of iterations and the last index.

$n := 0 .. N$

Calculate the change in gas bubble size for each time period and add to current ΔVol. The flow to the cylinders and the oil leakage makes the gas bubble bigger. The pump flow makes the gas bubble smaller.

$\Delta Vol_{n+1} := \max\left[\Delta Vol_n + \left(Q1(n \cdot \Delta t) + Qleak - 0.8 \cdot Qmax\right) \cdot \Delta t, 0\right]$

$\Delta V_{max} := \max(\Delta Vol) \cdot in^3$ $\Delta V_{max} = 0.026\,gal$ Find the max ΔV

Compute change from steady state gas volume V1. Note as oil flows out, the gas volume increases. This is why the max gas volume is $V1 + \Delta V$.

$$P_s \cdot V1^{\gamma} = P_{min} \cdot V2^{\gamma}$$

$$V2 = V1 + \Delta V_{max}$$

Substitute for V0 then Solve for the steady state gas volume V1.

$$P_s \cdot V1^{\gamma} = P_{min} \cdot \left(V1 + \Delta V_{max}\right)^{\gamma}$$

$$V1 := \frac{\Delta V_{max}}{\left(\frac{P_s}{P_{min}}\right)^{\frac{1}{\gamma}} - 1} \qquad V1 = 0.514\,\text{gal}$$

Calculate the maximum gas volume by adding the change in the oil volume to the steady state gas volume. The minimum accumulator volume is the same as the maximum gas volume. One may want to make the minimum accumulator volume just a little bigger for safety.

$$V1 + \Delta V_{max} = 0.54\,\text{gal}$$

Select accumulator by rounding up to next size.

Reference:C:\Documents and Settings\Peter\My Documents\mcd\hydraulics\AccumulatorSizeTable.xmcd(R)

$$V0 := \text{AccumSize}\left(V1 + \Delta V_{max}\right) \qquad V0 = 1\,\text{gal}$$

Accumulator size

Since the accumulator size is rounded up, V1 can be expanded to share half of the increase between V0-ΔV and the previous V1. Taking the average is arbitrary. One can set V1 to anything between V1 and V0-ΔV. The higher values will reduce the change in pressure.

$$V1 := \frac{V0 - \Delta V_{max} + V1}{2} \qquad V1 = 0.744\,\text{gal}$$

Set the new V1 to the average of V0-ΔV and V1

Pre charge the accumulator. Calculate the pre charge pressure P0.

Solve for pre charge pressure P0. Take into account temperature pre-charge and running temperature. The worst case would be pre-charging at operating temperature and running at room temperature.

$T0 := 75$

Pre-charge at room temperature

$T1 := 120$

Operational temperature

$$P0 := \frac{P_s \cdot V1}{V0}$$

Calculate pre-charge pressure at operations temperature. Use the ideal gas law because there is plenty of time for heat transfer.

$$P0 \cdot \frac{460 + T0}{460 + T1} = 1029.427$$

Pre-charge pressure compensated for the difference between room temperature and operational temperature.

$$V0 := V0 \cdot \frac{1}{gal} \qquad V1 := V1 \cdot \frac{1}{gal}$$

Make unitless for calculations. Mathcad can't raise volume to a fractional power γ

Graph Supply Pressure and Accumulator Volume As A Function of Time.

$$K := P0 \cdot V0^{\gamma}$$

Calculate K using pre charge values calculated above for pressure calculations below.

$$Vg_0 := \left(\frac{K}{P_s}\right)^{\frac{1}{\gamma}}$$

Initial accumulator gas volume calculated above

$$Ps_0 := P_s$$

Initial system pressure at steady state.

Calculate Pressure and Volume as a Function of Time.

Note that motion subtracts oil and lets the volume expand. It may be necessary to add a Qx function for each axis. The pump adds oil and makes the volume contract.

$n := 0 .. \frac{6 \cdot t_8}{\Delta t}$ Calculate number of iterations.

$$\begin{pmatrix} Ps_{n+1} \\ Vg_{n+1} \end{pmatrix} := \begin{bmatrix} \min\left[\frac{K}{\left(Vg_n\right)^{\gamma}}, P_s\right] \\ \max\left[Vg_n + \left(\frac{Q1(n \cdot \Delta t) + Qleak - Qp\left(Ps_n\right)}{231}\right) \cdot \Delta t, \left(\frac{K}{P_s}\right)^{\frac{1}{\gamma}}\right] \end{bmatrix}$$