Two Masses Connected by a Spring Simulation

The force on mass 0 as a function of the position of mass 0 and mass 1

$$F_0(x_0, x_1) = m_0 \cdot \frac{d^2}{dt^2} x_0 = K \cdot (x_1 - x_0 - L)$$

The force on mass 0 as a function of the position of mass 0 and mass 1.
Note that $F_0(x_0, x_1) = -F_1(x_0, x_1)$ because x_0 and x_1 are transposed.

$$F_1(x_0, x_1) = m_1 \cdot \frac{d^2}{dt^2} x_1 = K \cdot (x_0 - x_1 + L)$$

Spring constant in N/m

$$K := 25$$

Length of the spring in meters.   The spring can expand and contract.

$$L := 1$$

Mass 0 in Kg

$$m_0 := 1.725$$

Mass 1 in Kg

$$m_1 := 1$$

Reduced mass in Kg. The reduced mass isn't import to the simulation but it is interesting to see that the mass vibrates at a frequency determined by the reduced mass, not the total mass, and the spring constant.

$$\mu := \frac{m_0 \cdot m_1}{m_0 + m_1} \qquad \mu = 0.633$$

Predicted frequency of oscillation in rad/s and Hz

$$\omega := \sqrt{\frac{K}{\mu}} \qquad \omega = 6.284 \qquad HZ := \frac{\omega}{2 \cdot \pi} \qquad HZ = 1$$

Resulting period of oscillation in seconds.

$$T := \frac{1}{HZ} \qquad T = 1$$

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Compute the positions and velocities for the two masses

$$y_0 := \begin{pmatrix} 0.0 \\ 1 \\ 1.5 \\ 0 \end{pmatrix}$$

Initial state of the two masses

$$D(t,y) := \left| \begin{array}{l} \begin{pmatrix} x_0 \\ x'_0 \\ x_1 \\ x'_1 \end{pmatrix} \leftarrow y \\ \begin{bmatrix} x'_0 \\ \dfrac{K}{m_0}\cdot\left(x_1 - x_0 - L\right) \\ x'_1 \\ \dfrac{K}{m_1}\cdot\left(x_0 - x_1 + L\right) \end{bmatrix} \end{array} \right.$$

Just to make things more obvious

The two second order differential equations are broken down into four first order differential equations that can be easily solved.

$$z := \text{rkfixed}\left(y_0, 0, 10, 1000, D\right)$$

Runge-Kutta fixed. Solve for the positions and velocities of the two masses.

$$COM := \frac{m_0 \cdot z^{\langle 1 \rangle} + m_1 \cdot z^{\langle 3 \rangle}}{m_0 + m_1}$$

Center of Mass as a function of time

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$z =$

	Time 0	x_0 1	x'_0 2	x_1 3	x'_1 4
0	0	0	1	1.5	0
1	0.01	0.01	1.072	1.499	-0.124
2	0.02	0.021	1.142	1.498	-0.244
3	0.03	0.033	1.21	1.495	-0.362
4	0.04	0.046	1.275	1.49	-0.475
5	0.05	0.059	1.338	1.485	-0.584
6	0.06	0.072	1.399	1.479	-0.688
7	0.07	0.087	1.456	1.471	-0.787
8	0.08	0.101	1.51	1.463	-0.88
9	0.09	0.117	1.561	1.454	-0.967
10	0.1	0.133	1.608	1.444	-1.048
11	0.11	0.149	1.651	1.433	-1.123
12	0.12	0.166	1.69	1.421	-1.19
13	0.13	0.183	1.725	1.409	-1.25
14	0.14	0.2	1.755	1.396	-1.303
15	0.15	0.218	1.782	1.383	-1.348
16	0.16	0.236	1.803	1.369	-1.386
17	0.17	0.254	1.82	1.355	-1.415
18	0.18	0.272	1.833	1.341	-1.436
19	0.19	0.291	1.84	1.327	-1.449
20	0.2	0.309	1.843	1.312	-1.454
21	0.21	0.327	1.841	1.297	-1.451
22	0.22	0.346	1.834	1.283	-1.439
23	0.23	0.364	1.823	1.269	-1.42
24	0.24	0.382	1.807	1.255	-1.392
25	0.25	0.4	1.786	1.241	-1.356
26	0.26	0.418	1.761	1.228	-1.312
27	0.27	0.435	1.731	1.215	-1.261
28	0.28	0.453	1.697	1.202	-1.202
29	0.29	0.469	1.658	1.191	-1.136
30	0.3	0.486	1.616	1.18	-1.063
31	0.31	0.502	1.57	1.169	-0.983
32	0.32	0.517	1.52	1.16	-0.897
33	0.33	0.532	1.466	1.152	-0.804
34	0.34	0.546	1.41	1.144	-0.707
35	0.35	0.56	1.35	1.137	-0.603
36	0.36	0.573	1.287	1.132	-0.495
37	0.37	0.586	1.222	1.128	-0.383
38	0.38	0.598	1.154	1.124	-0.266
39	0.39	0.609	1.085	1.122	-0.146
40	0.4	0.619	1.013	1.121	-0.023
41	0.41	0.629	0.94	1.122	0.103
42	0.42	0.638	0.866	1.123	0.23
43	0.43	0.647	0.791	1.126	0.36

The data table is the resulting positions and velocities for mass 0 and mass 1 as a function of time. Mathcad allows one to scroll through the data.

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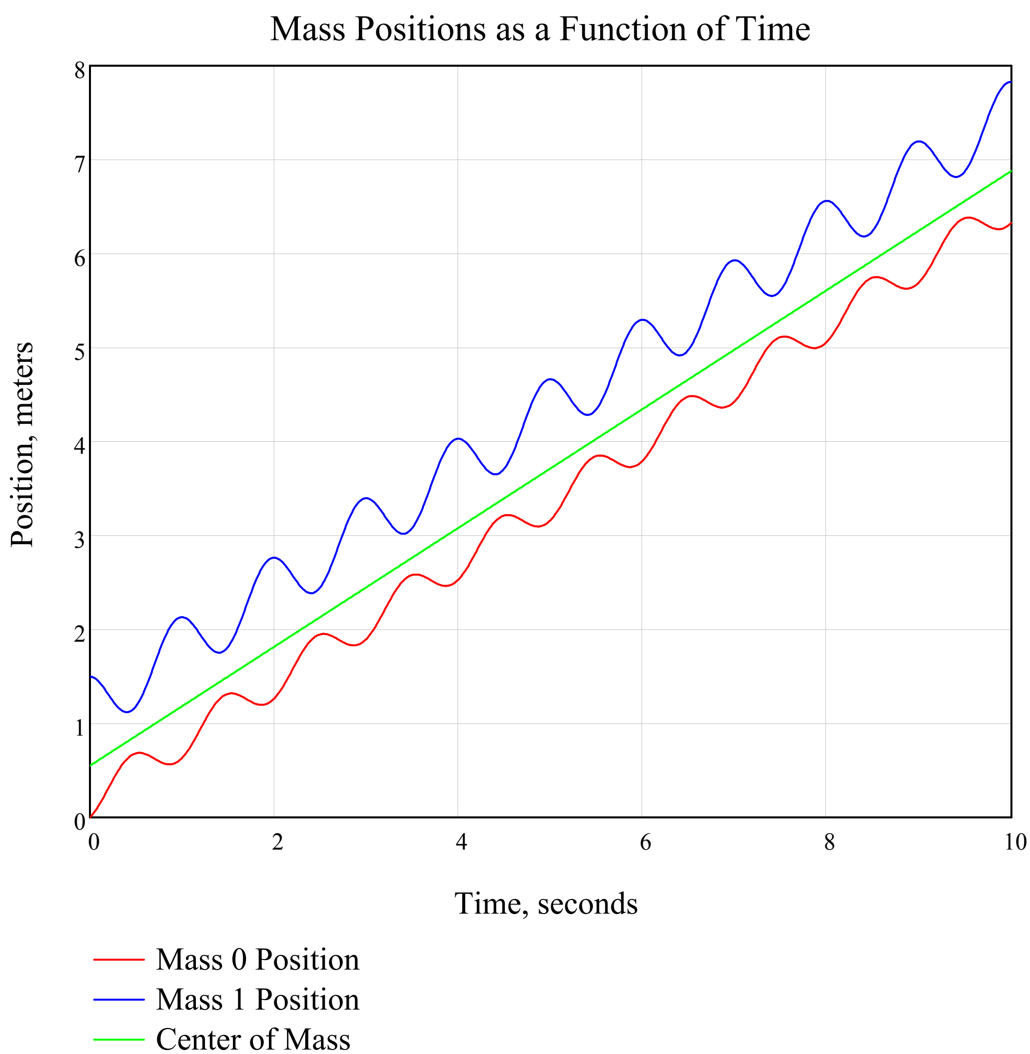

What is interesting is that the two mass and spring travel at the speed determined by the conservation of momentum for the two masses but they vibrate at a frequency determined by the reduced mass.